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Subject Name: **Strength of materials**

Subject Code: **ME-3002**

Semester: **3rd**

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Strength of Material (ME-3002) Class Notes

UNIT III

shear force diagram can be constructed from the loading diagram of the beam an order to draw this first the reactions must be determined & Then the vertical components of forces and reactions are successive + summed from the left end of the beam to preserve the mathematical sign conventions adopted& The shear at a section is simple+ equal to the sum of all the vertical forces to the left of the section&-hen the successive summation process is used' the shear force diagram should end up with the previous calculated shear .reaction at right end of the beam& /o shear force acts through the beam (must be +and the last vertical force or reaction & *f the shear force diagram closes in this fashion then it gives an important chec0 on mathematical calculations &The bending moment diagram is obtained b+ proceeding continuous along the length of beam from the left hand end and summing up the areas of shear force diagrams giving due regard to sign & The process of obtaining the moment diagram from the shear force diagram b+ summation is + the same as that for drawing shear force diagram from load diagram also be observed that a constant shear force produces a uniform change in the bending moment resulting in straight line in the moment diagram& *f no shear force exists along a certain portion of a beam' then it indicates that there is no change in moment ta0es place& *t ma+ also further observe that dm1 therefore' from the fundamental theorem of calculus the minimum moment occurs where the shear is zero& *n order to chec0 the validate of the bending moment diagram' the terminal conditions for the moment must be satisfied& *f the end is free or pinned' the computed sum must be equal to zero & *f the end is built in' the moment computed b+ the summation must be equal to the one calculated initial

+ for the reaction& These conditions must always be satisfied

The advantage of plotting a variation of shear force F and bending moment M in a beam as a function of 'x' measured from one end of the beam is that it becomes easier to determine the maximum absolute value of shear force and bending moment.

Further, the determination of value of M as a function of 'x' becomes of paramount importance so as to determine the value of deflection of beam subjected to a given loading.

Construction of shear force and bending moment diagrams:

A shear force diagram can be constructed from the loading diagram of the beam. In order to draw this, first the reactions must be determined always. Then the vertical components of forces and reactions are successively summed from the left end of the beam to preserve the mathematical sign conventions adopted. The shear at a section is simply equal to the sum of all the vertical forces to the left of the section.

When the successive summation process is used, the shear force diagram should end up with the previously calculated shear (reaction at right end of the beam. No shear force acts through the beam just beyond the last vertical force or reaction. If the shear force diagram closes in this fashion, then it gives an important check on mathematical calculations.

The bending moment diagram is obtained by proceeding continuously along the length of beam from the left hand end and summing up the areas of shear force diagrams giving due regard to

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sign. The process of obtaining the moment diagram from the shear force diagram by summation is exactly the same as that for drawing shear force diagram from load diagram.

It may also be observed that a constant shear force produces a uniform change in the bending moment, resulting in straight line in the moment diagram. If no shear force exists along a certain portion of a beam, then it indicates that there is no change in moment takes place. It may also further observe that dm/dx= F therefore, from the fundamental theorem of calculus the maximum or minimum moment occurs where the shear is zero. In order to check the validity of the bending moment diagram, the terminal conditions for the moment must be satisfied. If the end is free or pinned, the computed sum must be equal to zero. If the end is built in, the moment computed by the summation must be equal to the one calculated initially for the reaction. These conditions must always be satisfied.

From a design point of view, it is often necessary to know the maximum and minimum internal loads in a structure and where they are located.

If the shear force and bending moment are calculated and graphed, then the maximum and minimum of each are easily identified and located.

The benefits of drawing a variation of shear force and bending moment in a beam as a function of ‘x' measured from one end of the beam is that it becomes easier to determine the maximum absolute value of shear force and bending moment. The shear force and bending moment diagram gives a clear picture in our mind about the variation of SF and BM throughout the entire section of the beam.

Further, the determination of value of bending moment as a function of ‘x' becomes very important so as to determine the value of deflection of beam subjected to a given loading where we will use the formula,

$$EI\frac{d^2y}{dx^2} = M_x.$$

Notation and sign convention

Shear force (V)
Positive Shear Force

A shearing force having a downward direction to the right hand side of a section or upwards to the left hand of the section will be taken as ‘positive’. It is the usual sign conventions to be followed for the shear force. In some book followed totally opposite sign convention.

Negative Shear Force

A shearing force having an upward direction to the right hand side of a section or downwards to the left hand of the section will be taken as ‘negative’

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Bending Moment (M)

Positive Bending Moment

A bending moment causing concavity upwards will be taken as ‘positive’ and called as sagging bending moment.

Way to remember sign convention

Remember in the *Cantilever beam both Shear force and BM are negative (–ive).*

Relation between S.F (V_x), B.M. (M_x) & Load (w)

The value of the distributed load at any point in the beam is equal to the slope of the shear force curve.

(Note that the sign of this rule may change depending on the sign convention used for the external distributed load).

The value of the shear force at any point in the beam is equal to the slope of the bending moment curve.

Procedure for drawing shear force and bending moment diagram

Construction of shear force diagram

From the loading diagram of the beam constructed shear force diagram.

First determine the reactions.

Then the vertical components of forces and reactions are successively summed from the left end of the beam to preserve the mathematical sign conventions adopted. The shear at a section is simply equal to the sum of all the vertical forces to the left of the section.

The shear force curve is continuous unless there is a point force on the beam. The curve then “jumps” by the magnitude of the point force (+ for upward force).

When the successive summation process is used, the shear force diagram should end up with the previously calculated shear (reaction at right end of the beam). No shear force acts through the beam just beyond the last vertical force or reaction. If the shear force diagram closes in this fashion, then it gives an important check on mathematical

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calculations. i.e. The shear force will be zero at each end of the beam unless a point force is applied at the end.

Construction of bending moment diagram

The bending moment diagram is obtained by proceeding continuously along the length of beam from the left hand end and summing up the areas of shear force diagrams using proper sign convention.

The process of obtaining the moment diagram from the shear force diagram by summation is exactly the same as that for drawing shear force diagram from load diagram.

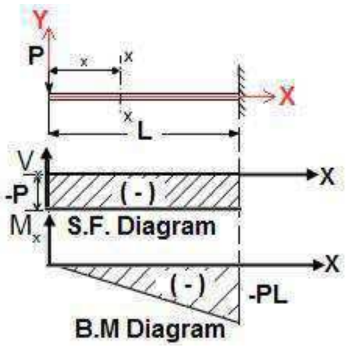
The bending moment curve is continuous unless there is a point moment on the beam. The curve then "jumps" by the magnitude of the point moment (+ for CW moment).

We know that a constant shear force produces a uniform change in the bending moment, resulting in straight line in the moment diagram. If no shear force exists along a certain portion of a beam, then it indicates that there is no change in moment takes place. We also know that dM/dx= Vx therefore, from the fundamental theorem of calculus the maximum or minimum moment occurs where the shear is zero.

The bending moment will be zero at each free or pinned end of the beam. If the end is built in, the moment computed by the summation must be equal to the one calculated initially for the reaction.

Shear force:

At a section a distance x from free end consider the forces to the left, then ***(Vx) = - P (for all values of x)*** negative in sign i.e. the shear force to the left of the x-section are in downward direction and therefore negative.



Bending Moment:

Taking moments about the section gives (obviously to the left

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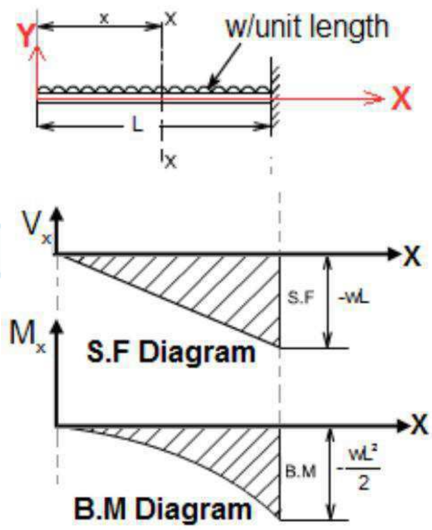
of the section) ***Mx = -P.x*** (negative sign means that the S.F and B.M diagram moment on the left hand side of the portion is in the

Anticlockwise direction and is therefore taken as negative according to the sign convention) so that the ***maximum*** bending moment occurs at the fixed end i.e. ***Mmax = -PL***(at x = L)

A Cantilever beam with uniformly distributed load over the whole length

When a cantilever beam is subjected to a uniformly distributed load whose intensity is given w /unit length.

Shear force:



Consider any cross-section XX which is at a distance of x from the free end. If we just take the resultant of all the forces on the left of the X-section, then

Vx = -w.x for all values of 'x'.

At x = 0, Vx = 0

At x = L, Vx = -wL (i.e. Maximum at fixed end)

Plotting the equation **Vx = -w.x**, we get a straight line because it is a equation of a straight line y (Vx) = m(- w) .x

Bending Moment:

Bending Moment at XX is obtained by treating the load to the

left of XX as a concentrated load of the same value (w.x) S.F and B.M diagram acting through the centre of gravity at x/2.

Therefore *the variation of bending moment is according to* ***parabolic law.***

The extreme values of B.M would be

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at x = 0, $M_x = 0$

and x = L, $M_x = -\frac{wL^2}{2}$

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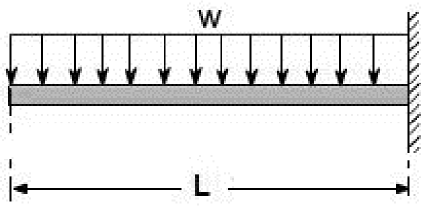
Maximum bending moment, $M_{max} = \frac{wL^2}{2}$ at fixed end

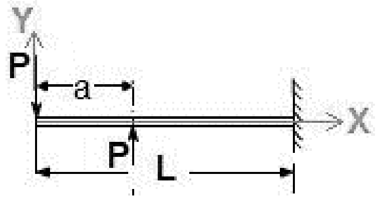
Another way to describe a cantilever beam with uniformly distributed load (UDL) over it's whole length.



(iii) A Cantilever beam loaded as shown below draw its S.F and B.M diagram

In the region 0 < x < a

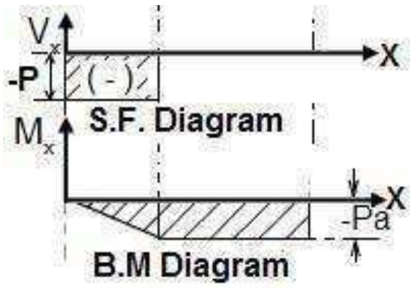
Following the same rule as followed previously, we get



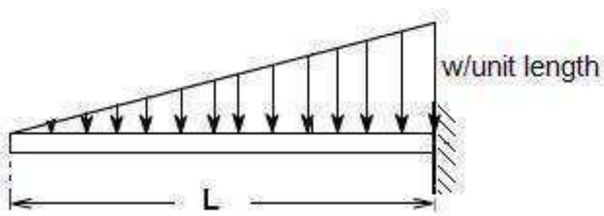
$V_x = -P$; and $M_x = -P.x$

In the region a < x < L

$V_x = -P+P=0$; and $M_x = -P.x + P(x-a) = P.a$



Cantilever beam carrying uniformly varying load from zero at free end and w/unit length at the fixed end



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Consider any cross-section XX which is at a distance of x from the free end.

Shear force (V_x) = area of ABC (load triangle)

∴ The shear force variation is parabolic. at x = 0, $V_x = 0$

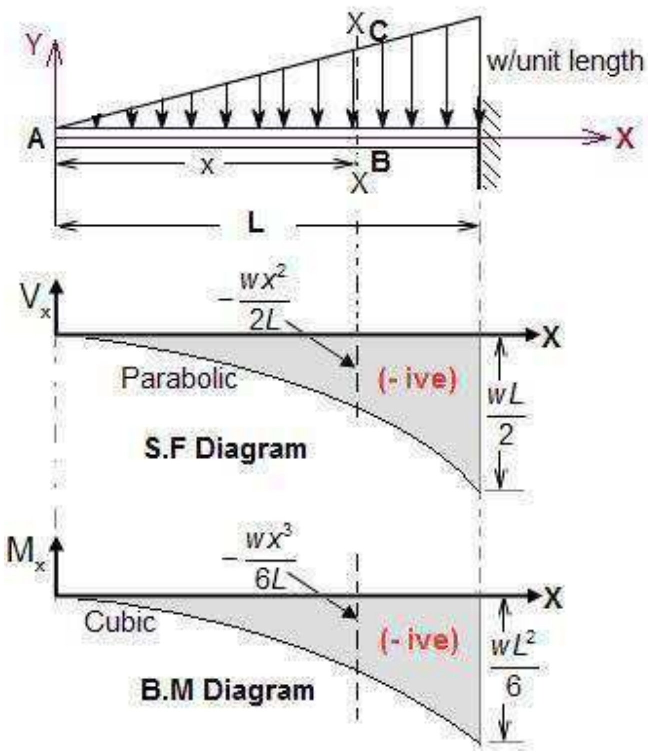
at x = L, $V_x = -\frac{WL}{2}$ i.e. Maximum Shear force (V_{max}) =

Bending moment (M_x) = load × distance from centroid

∴ The bending moment variation is cubic.

at x= 0, $M_x = 0$

at x = L, $M_x = -\frac{wL^2}{6}$ i.e. Maximum Bending moment (M_{max}) = $\frac{wL^2}{6}$ at fixed end.



Alternative way : (Integration method)

We know that $\frac{d(V_x)}{dx} = -\text{load} = -\frac{w}{L}.x$

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$$\text{or } d(V_x) = -\frac{w}{L}.x.dx$$

Integrating both side

$$\int_0^{V_x} d(V_x) = -\int_0^x \frac{w}{L}.x.dx$$

$$\text{or } V_x = -\frac{w}{L}.\frac{x^2}{2}$$

Again we know that

$$\frac{d(M_x)}{dx} = V_x = -\frac{wx^2}{2L}$$

$$\text{or } d(M_x) = -\frac{wx^2}{2L}dx$$

Integrating both side we get (at x=0, M_x =0)

$$\int_0^{M_x} d(M_x) = -\int_0^x \frac{wx^2}{2L}.dx$$

$$\text{or } M_x = -\frac{w}{2L} \times \frac{x^3}{3} = -\frac{wx^3}{6L}$$

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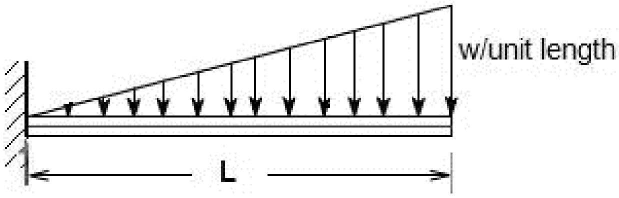
A Cantilever beam carrying gradually varying load from zero at fixed end and w/unit length at the free end



Considering equilibrium we get, $M_A = \frac{wL^2}{3}$ and Reaction (RA) = $\frac{wL}{2}$

considering any cross-section XX which is at a distance of x from the fixed end.

At this point load (W_x) = $\frac{W}{L}.x$

Shear force (Vx) = R_A – area of triangle ANM

$$= \frac{wL}{2} - \frac{1}{2}\cdot\frac{w}{L}.x.x = +\frac{wL}{2} - \frac{wx^2}{2L}$$

∴ the shear force variation is parabolic.

At x = 0, $V_x = +\frac{wL}{2}$ i.e. Maximum shear force, $V_{max} = +\frac{wL}{2}$ at x = L, $V_x = 0$

Bending moment (Mx) =RA .x - MA

Take a section at a distance x from the left support. This section is applicable for any value of x just to the left of the applied force P. The shear, remains constant and is +P. The bending moment varies linearly from the support, reaching a maximum of +Pa.

A section applicable anywhere between the two applied forces. Shear force is not necessary to maintain equilibrium of a segment in this part of the beam. Only a constant bending moment of +Pa must be resisted by the beam in this zone.

Such a state of bending or flexure is called ***pure bending.***

Shear and bending-moment diagrams for this loading condition are shown below.

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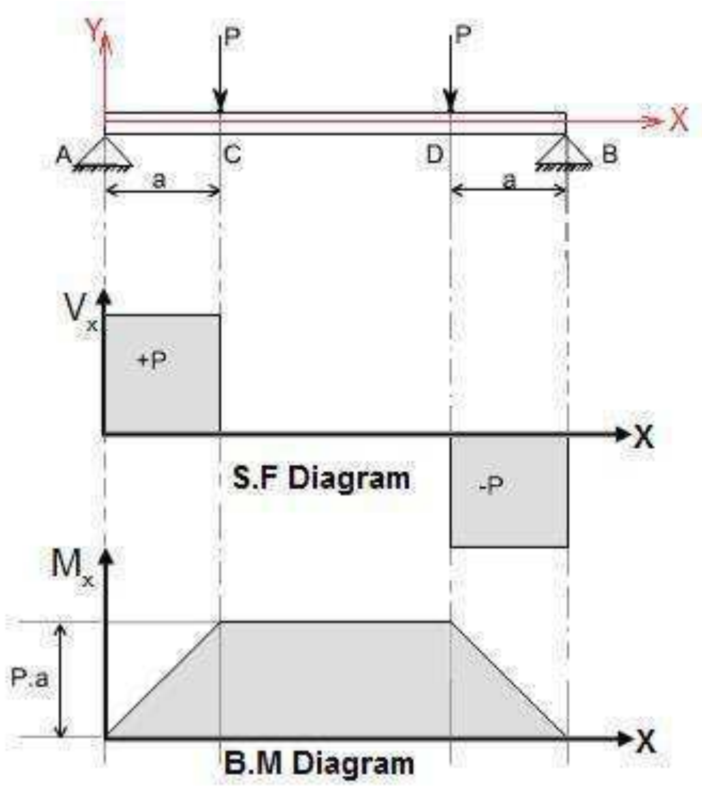
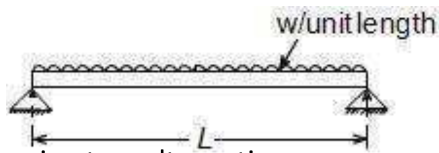


A Simply supported beam with a uniformly distributed load (UDL) throughout its length



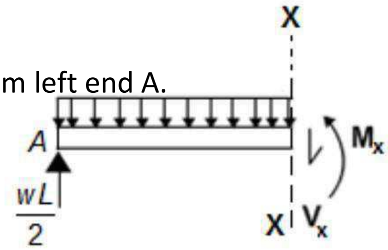
We will solve this problem by following two alternative ways.

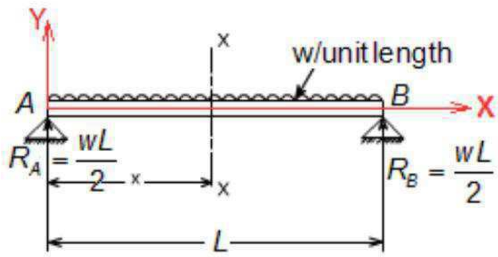
(a) By Method of Section

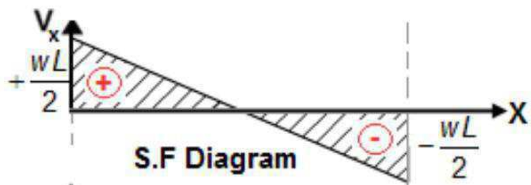
Considering equilibrium we get RA = RB = $\frac{wL}{2}$

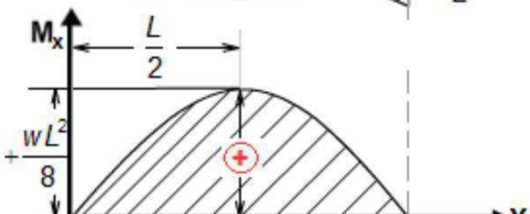
Now Consider any cross-section XX which is at a distance x from left end A.

Then the section view









Shear force: $V_x = \frac{wL}{2} - wx$

(i.e. S.F. variation is linear)

at x = 0, $V_x = \frac{wL}{}$

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2

at x = L/2, Vx = 0

at x = L, $V_x = -\frac{wL}{2}$

Bending moment: $M_x = \frac{wL}{2} . x - \frac{wx^2}{2}$

(i.e. B.M. variation is parabolic)

at x = 0, Mx = 0

at x = L, Mx = 0

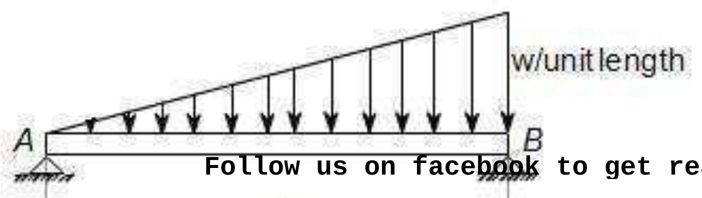
Now we have to determine maximum bending moment and its position.

For maximum B.M: $\frac{d(M_x)}{dx} = 0 \;\; i.e.\; V_x = 0 \;\; \because \frac{d(M_x)}{dx} = V_x$

or $\frac{wL}{2} - wx = 0 \;\; or \;\; x = \frac{L}{2}$

Therefore, maximum bending moment, $M_{max} = \frac{wL^2}{\;}$ **at x = L/2**

A Simply supported beam with a gradually varying load (GVL) zero at one end and w/unit length at other span.



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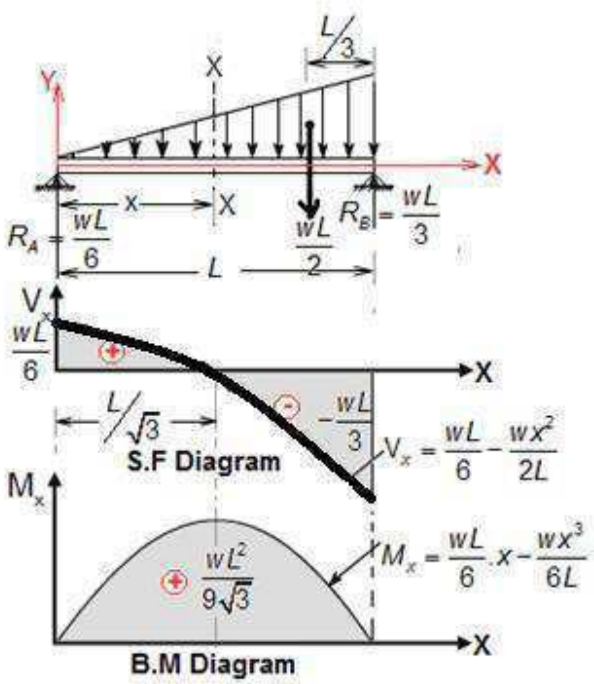
Consider equilibrium of the beam = 0.5 wL acting at a point C at a distance 2L/3 to the left end A.

$\sum M_B = 0$ gives

Similarly $\sum M_A = 0$ gives $R_B = \frac{wL}{3}$

The free body diagram of section A - XX as shown below, Load at section XX, $(wx) = \frac{w}{L}x$



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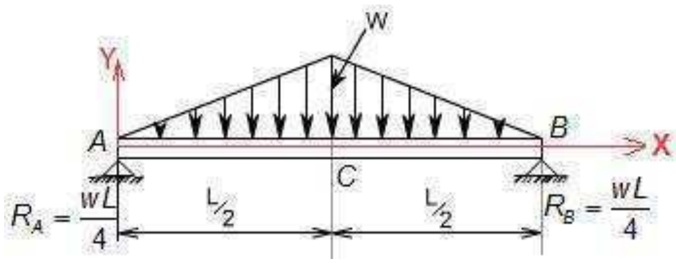
A Simply supported beam with a gradually varying load (GVL) zero at each end and w/unit length at mid span.



The free body diagram of section A –XX as shown below, load at section XX (wx) = $\frac{2w}{L}.x$

The resultant of that part of the distributed load which acts on this free body is = $\frac{1}{2}.x.\frac{2w}{L}.x = \frac{wx^2}{L}$ applied at a point, distance x/3 from section XX.

Shear force (Vx):

In the region 0 < x < L/2

$$(V_x) = R_A - \frac{wx^2}{L} = \frac{wL}{4} - \frac{wx^2}{L}$$

Therefore the variation of shear force is parabolic.

at x = 0, $\quad V_x = \frac{wL}{4}$

at x = L/4, $\quad V_x = 0$

In the region of L/2 < x < L

The Diagram will be Mirror image of AC.

Bending moment (Mx):

In the region 0 < x < L/2

$wL \quad 1 \quad 2wx.(x/3) = wL \quad wx^3$

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$M_x = \frac{\quad}{4}.x - \frac{\quad}{2}.x.\frac{\quad}{L} \qquad \frac{\quad}{4} - \frac{\quad}{3L}$

The variation of BM is cubic

at x = 0, Mx = 0

at x = L/2, $M_x = \frac{wL^2}{12}$

In the region L/2 < x < L

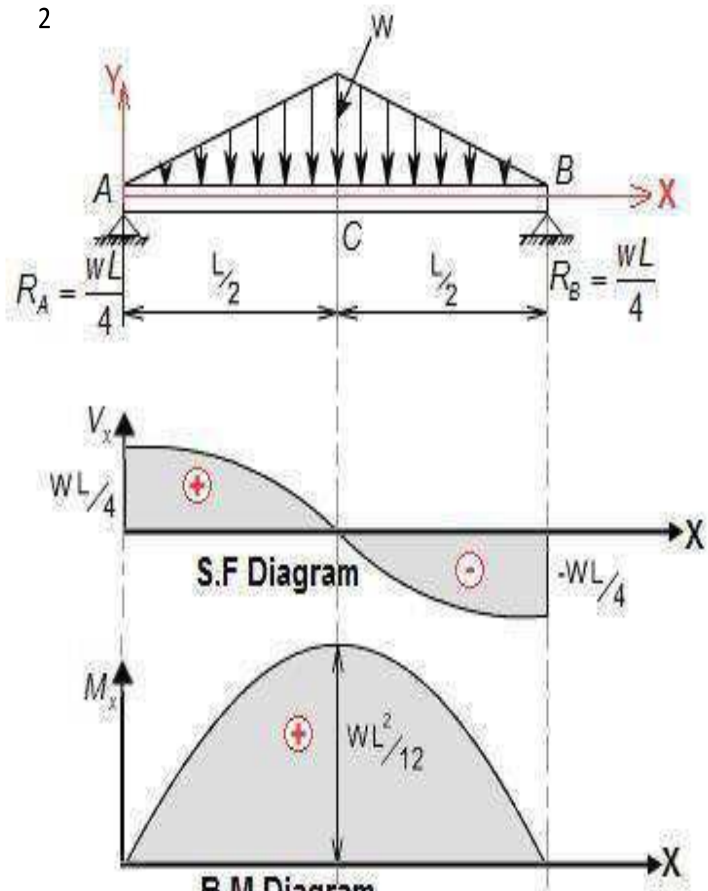
BM diagram will be mirror image of AC.

For maximum bending moment

$\frac{d(M_x)}{dx} = 0$ i.e. $V_x = 0$ $\qquad \because \frac{d(M_x)}{dx} = V_x$

or $\frac{wL}{4} - \frac{wx^2}{L} = 0$ or $x = \frac{L}{2}$

and $M_{max} = \frac{wL^2}{12}$



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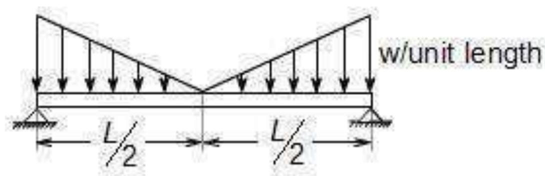
A Simply supported beam with a gradually varying load (GVL) zero at mid span and w/unit length at each end.



Bending Moment diagram of Statically Indeterminate beam

Beams for which reaction forces and internal forces **cannot** be found out from static equilibrium equations alone are called statically indeterminate beam. This type of beam requires deformation equation in addition to static equilibrium equations to solve for unknown forces.

Statically determinate - Equilibrium conditions sufficient to compute reactions.

Statically indeterminate - Deflections (Compatibility conditions) along with equilibrium equations should be used to find out reactions.

Load and Shear Force diagram from Bending Moment diagram

If S.F. Diagram for a beam is given, then

If S.F. diagram consists of rectangle then the load will be point load

If S.F diagram consists of inclined line then the load will be UDL on that portion

If S.F diagram consists of parabolic curve then the load will be GVL

If S.F diagram consists of cubic curve then the load distribute is parabolic.

After finding load diagram we can draw B.M diagram easily.

If B.M Diagram for a beam is given, then

If B.M diagram consists of vertical line then a point BM is applied at that point.

If B.M diagram consists of inclined line then the load will be free point load

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If B.M diagram consists of parabolic curve then the load will be U.D.L.

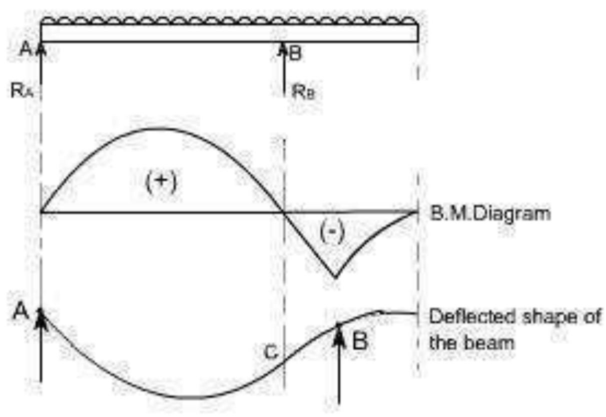
If B.M diagram consists of cubic curve then the load will be G.V.L.

If B.M diagram consists of fourth degree polynomial then the load distribution is parabolic.

Point of Contraflexure

In a beam if the bending moment changes sign at a point, the point itself having zero bending moment, the beam changes curvature at this point of zero bending moment and this point is called the point of contra flexure.

Consider a loaded beam as shown below along with the B.M diagrams and deflection diagram.



In this diagram we noticed that for the beam loaded as in this case, the bending moment diagram is partly positive and partly negative. In the deflected shape of the beam just below the bending moment diagram shows that left hand side of the beam is 'sagging' while the right hand side of the beam is 'hogging'.

The point C on the beam where the curvature changes from sagging to hogging is a point of contraflexure.

There can be more than one point of contraflexure in a beam.

Assumptions in Simple Bending Theory

Beams are initially straight

The material is homogenous and isotropic i.e. it has a uniform composition and its mechanical properties are the same in all directions

The stress-strain relationship is linear and elastic

Young's Modulus is the same in tension as in compression

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Sections are symmetrical about the plane of bending

Sections which are plane before bending remain plane after bending

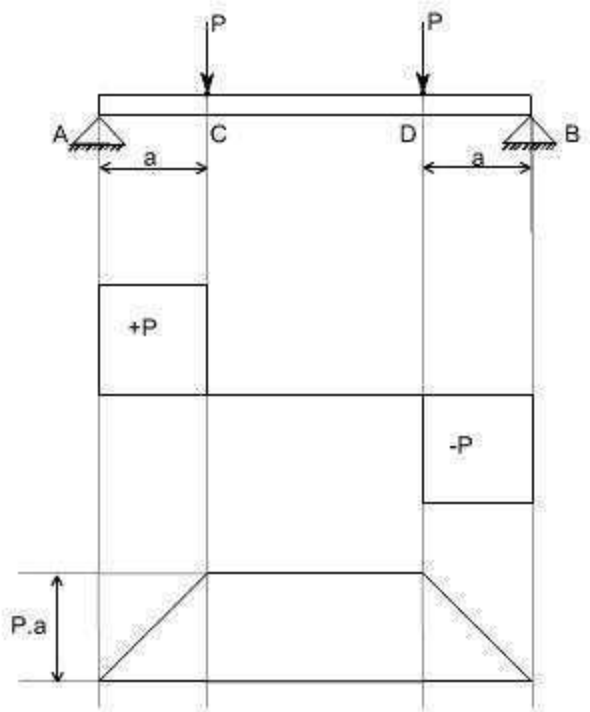
Non-Uniform Bending

In the case of non-uniform bending of a beam, where bending moment varies from section to section, there will be shear force at each cross section which will induce shearing stresses

Also these shearing stresses cause warping (or out-of plane distortion) of the cross section so that plane cross sections do not remain plane even after bending.

Beams under shear stress:

Theory which has been discussed earlier, while we discussed the bending stresses in beams was for the case of pure bending i.e. constant bending moment acts along the entire length of the beam.



Take a consideration of the beam AB transversely loaded as shown in the figure above. Together with shear force and bending moment diagrams we note that the middle potion CD of the beam is free from shear force and that it's bending moment. M = Pxa is uniform between the portion C and D. This condition is called the pure bending condition.

Since shear force and bending moment are related to each other F= dM/dX (eq) therefore if the shear force changes than there will be a change in the bending moment also, and then this won't be the pure bending.

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Bending Equation:

Hence one can conclude from the pure bending theory was that the shear force at each X-section is zero and the normal stresses due to bending are the only ones produced.

In the case of non-uniform bending of a beam where the bending moment varies from one X-section to another, there is a shearing force on each X-section and shearing stresses are also induced in the material. The deformation associated with those shearing stresses causes " warping " of the x-section so that the assumption which we assumed while deriving the relation $\frac{\sigma}{y}=\frac{M}{I}=\frac{E}{R}$ that the plane cross-section after bending remains plane is violated. Now due to warping the plane cross=section before bending do not remain plane after bending. This complicates the problem but more elaborate analysis shows that the normal stresses due to bending, as calculated from the equation. $\frac{\sigma}{y}=\frac{M}{I}=\frac{E}{R}$

So the above equation gives the distribution of stresses which are normal to the cross-section that is in x-direction or along the span of the beam are not greatly altered by the presence of these shearing stresses. Thus, it is justifiable to use the theory of pure bending in the case of non-uniform bending and it is accepted practice to do so and similarly non bending equation and cases can be solved.

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